

In a set of N data points x_i the following statistical properties are defined:

- mean $m = \frac{\sum x_i}{N}$
- root mean square $RMS(x_i) = \sqrt{\frac{\sum x_i^2}{N}}$
- deviation $d_i = x_i - m$
- variance $v = RMS^2(d_i) = \frac{\sum (x_i^2) - 2m \sum x_i + m^2}{N} = RMS^2(x_i) - m^2$
- standard deviation $\mathbf{s} = \sqrt{v}$

The norm of a pair x_i and x_j is defined as

$$X_{ij} = \sqrt{x_i^2 + x_j^2} = \sqrt{2m^2 + 2m(d_i + d_j) + d_i^2 + d_j^2} \quad (1)$$

The sum of the squares of L norms is

$$\begin{aligned} \sum_L X_{ij}^2 &= \sum_L (2m^2 + 2md_i + 2md_j + d_i^2 + d_j^2) \\ &= 2Lm^2 + 2m \sum_L d_i + 2m \sum_L d_j + \sum_L d_i^2 + \sum_L d_j^2 \end{aligned} \quad (2)$$

For values $L \approx N$, the sums of deviations approach zero while the sums of squares of deviations approach the value $L\mathbf{s}^2$. Expression (2) can then be simplified to

$$\sum_L X_{ij}^2 = 2Lm^2 + 2L\mathbf{s}^2 \Rightarrow \frac{1}{L} \sum_L X_{ij}^2 \quad (\equiv RMS^2(X_{ij})) = 2m^2 + 2\mathbf{s}^2 \quad (3)$$

Considering the two cases $\mathbf{s} = 0$ and $m = 0$, it becomes clear from (3) that the mean and standard deviation of the norms follow from the following relations:

$$\begin{aligned} M^2(X_{ij}) &= 2m^2(x_i) \Rightarrow M = \sqrt{2} m \\ V(X_{ij}) &= 2v(x_i) \Rightarrow \mathbf{s}(X) = \sqrt{2} \mathbf{s}(x) \end{aligned} \quad (4)$$

We have another data set y_i with norms Y_{ij} , and it has been observed that the ratios $R = \frac{X_{ij}}{Y_{ij}}$ show

a well-defined mean value and standard deviation. From this we would like to derive the behaviour of

the mean value and standard deviation of the ratios $r = \frac{x_{ij}}{y_{ij}}$.

Using the expressions (4) we can write

$$M(R) \equiv \frac{M(X)}{M(Y)} = \frac{\sqrt{2}m(x)}{\sqrt{2}m(y)} = \frac{m(x)}{m(y)} \equiv m(r) \quad (5)$$

and

$$\mathbf{s}(R) \equiv \frac{\mathbf{s}(X)}{\mathbf{s}(Y)} = \frac{\sqrt{2}\mathbf{s}(x)}{\sqrt{2}\mathbf{s}(y)} = \frac{\mathbf{s}(x)}{\mathbf{s}(y)} \equiv \mathbf{s}(r) \quad (6)$$

Conclusion: the mean and standard deviation of the ratios r are the same as those of the ratios R .